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# Stationary axisymmetric electromagnetic fields in the Kerr metric

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**Abstract.** A procedure for calculating the Debye potential corresponding to a stationary axisymmetric distribution of charges and currents in the Kerr metric is given. The electromagnetic potential is derived by differentiation. The potential of a point charge on the symmetry axis and the value of the charge resulting from the accretion in the case of a charged current loop situated in the equatorial plane are determined with this formalism.

## 1. Introduction

The generation of an electromagnetic field by stationary or dynamical sources in the Schwarzschild and the Kerr background has been a subject of considerable investigation. This paper is devoted to solving the field for all stationary axisymmetric distributions of charges and currents in the Kerr metric.

We have shown (Linet 1976) that it is possible to determine the electrostatics and the magnetostatics in the Schwarzschild metric. We have given the electrostatic potential of a point charge at rest in algebraic form and a formula allowing us to find the magnetic potential in integral form. With this method in the case of a current loop a numerical calculation is possible (Damour *et al* 1978). Another method was to give these solutions in the form of series of multipoles (Cohen and Wald 1971, Hanni and Ruffini 1973, Petterson 1974, Bicak and Dvorak 1977).

Recently we have determined explicitly (Linet 1977a) the stationary axisymmetric Green function of the equation of Teukolsky (1973). It means that the complex components $\phi_0$ and $\phi_2$ of the electromagnetic field, in Newman and Penrose notation, can be determined from stationary axisymmetric sources. Yet the other component $\phi_1$ and the two components $A_t$ and $A_\phi$ of the electromagnetic potential are not explicitly calculated. We shall see that this is possible by using the Debye potential introduced by Cohen and Kegeles (1974).

In § 2 we shall summarise the Maxwell equations for a stationary axisymmetric field in the Kerr metric, and we shall show how to derive the electromagnetic potential from the Debye potential. The main result shall be given in § 3, where we shall determine the Debye potential corresponding to the Green function of the Teukolsky equation for $\phi_0$. We shall discuss also the monopole field. Thus we have a procedure for calculating fields generated from stationary axisymmetric sources. Another method was to give these solutions in the form of series of multipoles (Cohen *et al* 1974, Chitre and Vishveshwara 1975, Petterson 1975, King 1976, Bicak and Dvorak 1976, Znajek

1978). In § 4 we shall use this formalism to find easily the potential of a point charge on the symmetry axis given by Léauté (1977) (the electromagnetic field has been given also by Misra (1977)). The case of a charged current loop situated in the equatorial plane shall be examined. We shall give the charge accretion (Linet 1977b) and the behaviour of the electromagnetic field when the radius of the loop tends to the horizon.

## 2. Maxwell equation in the Kerr metric

The Kerr space–time is characterised by the two parameters $M$ and $a$ with $a \leqslant M$. In the Boyer and Lindquist coordinates the Kerr metric $g_{\mu\nu}$ is

$$\begin{aligned} ds^2 = \left(1-\frac{2Mr}{\Sigma}\right) dt^2 - \frac{\Sigma}{\Delta(r)} dr^2 - \Sigma\, d\theta^2 + \frac{4aMr \sin^2\theta}{\Sigma} dt\, d\phi \\ - \sin^2\theta \left(r^2 + a^2 + \frac{2a^2 Mr}{\Sigma}\sin^2\theta\right) d\phi^2, \end{aligned} \tag{1}$$

with $\Delta(r) = r^2 - 2Mr + a^2$ and $\Sigma = r^2 + a^2\cos^2\theta$. The Schwarzschild metric is obtained by putting $a = 0$ in (1).

From the practical point of view, we must use the null tetrad formalism to solve the Maxwell equation in the background metric (1). Following standard convention, the components of the Kinnersley tetrad are

$$\begin{aligned} l^\mu &: \left(\frac{r^2+a^2}{\Delta(r)}, 1, 0, \frac{a}{\Delta(r)}\right), \\ n^\mu &: \frac{1}{2\Sigma}(r^2+a^2, -\Delta(r), 0, a), \\ m^\mu &: \frac{1}{\sqrt{2}(r+ia\cos\theta)}\left(ia\sin\theta, 0, 1, \frac{i}{\sin\theta}\right). \end{aligned} \tag{2}$$

Then the electromagnetic field is described by three tetrad components

$$\phi_0 = F_{\mu\nu} l^\mu m^\nu, \qquad \phi_1 = \tfrac{1}{2} F_{\mu\nu}(l^\mu n^\nu + \bar{m}^\mu m^\nu), \qquad \phi_2 = F_{\mu\nu}\bar{m}^\mu n^\nu. \tag{3}$$

By the method of Teukolsky (1973) one can derive from the Maxwell equation

$$\nabla_\rho F^{\rho\mu} = 4\pi J^\mu, \qquad \partial_\rho F_{\lambda\mu} + \partial_\lambda F_{\mu\rho} + \partial_\mu F_{\rho\lambda} = 0, \tag{4}$$

where $J^\mu$ is the current density, a decoupled equation for the quantities

$$\psi_1 = \phi_0, \qquad \psi_{-1} = (r - ia\cos\theta)^2 \phi_2. \tag{5}$$

In the case of stationary axisymmetric solutions we have

$$\psi_{-1} = -\tfrac{1}{2}\Delta(r)\psi_1, \tag{6}$$

and $\psi_s$, $s = \pm 1$ satisfies the equation

$$-\Delta(r)\frac{\partial^2}{\partial r^2}\psi_s - 2(s+1)(r-M)\frac{\partial \psi_s}{\partial r} - \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\psi_s\right) + (s^2\cot^2\theta - s)\psi_s = 4\pi T_s \Sigma, \tag{7}$$

where the term $T_s$ is expressed in terms of $J^\mu$ and its derivatives following the

Teukolsky equation. The source is assumed to be situated outside the outer horizon $r_+ = M + (M^2 - a^2)^{1/2}$.

For a source-free Maxwell field, Cohen and Kegeles (1974) have shown that the components $\phi_0$ and $\phi_1$ can be generated by differentiation of a Debye potential:

$$\phi_0 = -\partial^2 \bar{\psi}/\partial r^2, \tag{8}$$

$$\phi_1 = -\frac{1}{\sqrt{2}(r - ia\cos\theta)}\left[-\frac{1}{r - ia\cos\theta}\left(\frac{\partial}{\partial\theta} + \cot\theta + ia\sin\theta\frac{\partial}{\partial r}\right) + \frac{\partial^2}{\partial r\,\partial\theta} + \cot\theta\frac{\partial}{\partial r}\right]\bar{\psi}_s. \tag{9}$$

The Debye potential satisfies the same homogeneous equation as (7) for $s = -1$. It is useful to note that (7) has real coefficients; thus $\bar{\psi}$ satisfies the same equation.

On the other hand, the Maxwell field derives from a potential

$$F_{\mu\gamma} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{10}$$

It is more practical to introduce the tetrad components $A_l$, $A_n$ and $A_m$. In the case of stationary axisymmetric solutions it is easy to see that

$$A_n = \frac{\Delta(r)}{2\Sigma}A_l, \qquad A_m = -\frac{\mathrm{i}}{\sqrt{2}(r + \mathrm{i}a\cos\theta)}A, \tag{11}$$

where $A$ is a real quantity. Indeed

$$\begin{aligned} A_t &= (\Delta(r)A_l + a\sin\theta A)/\Sigma, \\ A_\phi &= -[(r^2 + a^2)\sin\theta A + a\sin^2\theta\Delta(r)A_l]/\Sigma. \end{aligned} \tag{12}$$

We remark that the component $\phi_0$ is obtained by differentiation from the tetrad components $A$ and $A_l$ in a simple way:

$$\phi_0 = -\frac{1}{\sqrt{2}(r + \mathrm{i}a\cos\theta)}\left(\mathrm{i}\frac{\partial A}{\partial r} + \frac{\partial A_l}{\partial\theta}\right). \tag{13}$$

Comparing (12) with (8), we determine the components $A$ and $A_l$ by differentiation of the Debye potential:

$$A = \sqrt{2}a\cos\theta\,\partial\,\mathrm{Re}\,\bar{\psi}/\partial r - \sqrt{2}(-r\,\partial\,\mathrm{Im}\,\bar{\psi}/\partial r + \mathrm{Im}\,\bar{\psi}) \tag{14}$$

$$A_l = \frac{-\sqrt{2}r}{\Delta(r)}\left(\frac{\partial}{\partial\theta}\mathrm{Re}\,\bar{\psi} + \frac{\cos\theta}{\sin\theta}\mathrm{Re}\,\bar{\psi}\right) + \frac{\sqrt{2}a}{\Delta(r)}\left(\cos\theta\frac{\partial}{\partial\theta}\mathrm{Im}\,\bar{\psi} + \frac{1}{\sin\theta}\mathrm{Im}\,\bar{\psi}\right). \tag{15}$$

The electromagnetic potential is given by equations (14) and (15) in the Lorentz gauge. We note that the equations of Chrzanowski (1975) giving the potential in the ingoing radiation gauge $A_l = 0$ from the Debye potential are not applicable in a stationary case.

## 3. Determination of the Debye potential

We want to determine the Debye potential corresponding to an electromagnetic field generated by a stationary axisymmetric source. As $\psi_1$ (or $\psi_{-1}$) specifies the solution of the Maxwell equation (except the monopole term), the first problem is to give the Green function of equation (7); that is a solution of (7) with the source term

$$T_s(r, \theta) = \delta(r - r_0)\delta(\cos\theta - \cos\theta_0)/\Sigma, \qquad \theta_0 \neq 0, \pi, \tag{16}$$

which is well-behaved at infinity and on the horizon $r_+$. The case $\theta_0=0$ or $\pi$ will be examined later because the following method breaks down for $\theta_0=0$ or $\pi$.

We introduce the function $f_s$ by

$$f_s(r,\theta)=(\sin\theta)^{-s}\psi_s(r,\theta), \tag{17}$$

and the new variables $\rho$ and $z$ by

$$z=(r-M)\cos\theta, \qquad \rho=\Delta^{1/2}(r)\sin\theta, \qquad r\geqslant r_+, \rho\geqslant 0. \tag{18}$$

Then $f_s$ satisfies the equation

$$\frac{\partial^2 f_s}{\partial z^2}+\frac{\partial^2 f_s}{\partial \rho^2}+\frac{(1+2s)}{\rho}\frac{\partial f_s}{\partial \rho}=4\pi\frac{\Delta^{s/2}(\rho_0,z_0)}{\rho_0^s}\frac{1}{\rho}\delta(\rho-\rho_0)\delta(z-z_0), \tag{19}$$

where $\rho_0$ and $z_0$ are defined from $r_0$ and $\theta_0$ with the help of equations (18).

The 'generalised axially symmetric potential' theory (GASP theory) is governed by the homogeneous equation (19) (Weinstein 1948). We give the Green function for $s=1$:

$$f_1(\rho,z)=-2\rho_0^2\int_0^\pi[(z-z_0)^2+\rho^2+\rho_0^2-2\rho\rho_0\cos\alpha]^{-3/2}\sin^2\alpha\,\mathrm{d}\alpha. \tag{20}$$

Taking account of (17) and (18), we obtain the Green function of (7) for $s=1$:

$$G_1(r,\theta;r_0,\theta_0)=-2\sin\theta\sin\theta_0\Delta(r_0)\int_0^\pi\frac{\sin^2\alpha}{\mathcal{R}^3}\mathrm{d}\alpha, \tag{21}$$

where we have put

$$\begin{aligned}\mathcal{R}^2=(r-M)^2+(r_0-M)^2-2(r-M)(r_0-M)\cos\theta\cos\theta_0\\ -(M^2-a^2)(\sin^2\theta+\sin^2\theta_0)-2\Delta^{1/2}(r)\Delta^{1/2}(r_0)\sin\theta\sin\theta_0\cos\alpha.\end{aligned}$$

For all stationary axisymmetric sources situated outside $\theta_0=0$ or $\pi$ we can find with (21) the component $\phi_0$ by the product of convolution $\phi_0=G_1{*}T_1\Sigma$ defined by

$$\phi_0(r,\theta)=\int_0^\infty\int_0^\pi G_1(r,\theta;r_0,\theta_0)T_1(r_0,\theta_0)(r_0^2+a^2\cos^2\theta_0)\sin\theta_0\,\mathrm{d}\theta_0\,\mathrm{d}r_0. \tag{22}$$

The Debye potential corresponding to (21) is calculated by using the function $g$ defined by

$$\bar{\psi}(r,\theta)=\Delta(r)\sin\theta g(r,\theta), \tag{23}$$

which satisfies, in the coordinates $\rho$ and $z$, the GASP equation for $s=1$. A basic property of this equation is that the values of $g$ on the axis $\rho=0$ allow us to determine the complete function in integral form:

$$g(\rho,z)=\frac{2}{\pi}\int_0^\pi g(0,z+\mathrm{i}\rho\cos\alpha)\sin^2\alpha\,\mathrm{d}\alpha. \tag{24}$$

But we remark

$$g(0,z)=\frac{1}{\Delta(r)}\frac{\bar{\psi}(r,\theta)}{\sin\theta}\bigg|_{\theta=0}. \tag{25}$$

Thus $g$ on the axis $\rho = 0$ can be calculated from $G_1$ by integrating it in the relation (8) at $\theta = 0$. We find

$$\left.\frac{\bar{\psi}(r,\theta)}{\sin\theta}\right|_{\theta=0} = \frac{\pi}{\sin\theta_0}\{[(r-M)^2+(r_0-M)^2-2(r-M)(r_0-M)\cos\theta_0 - (M^2-a^2)\sin^2\theta_0]^{1/2}-(r-M)+(r_0-M)\cos\theta_0\}+C_1(r-M)+C_2, \tag{26}$$

where $C_1$ and $C_2$ are two arbitrary constants, and the term between the square brackets tends to zero as $1/r$ when $r\to\infty$. Consequently

$$g(0,z)=\frac{\pi}{\sin\theta_0}\frac{1}{z^2-(M^2-a^2)}\{[(z-z_0)^2+\rho_0^2]^{1/2}-z+z_0\}+\frac{C_1 z + C_2}{z^2-(M^2-a^2)}. \tag{27}$$

Introducing the value of $g(0, z)$ from (27) in (24) we obtain $g(\rho, z)$, and by differentiation the electromagnetic potential.

The two constants $C_1$ and $C_2$ in (27) induce two Debye potentials giving the monopole field. If it is characterised by the total charge $E$, we can replace these two Debye potentials by one depending only on $E$:

$$\bar{\psi}_E(r,\theta) = -\frac{E}{\sqrt{2}}\frac{1-\cos\theta}{\sin\theta}. \tag{28}$$

## 4. Examples

### *4.1. Point charge on the symmetry axis*

The current density for a point charge $q$ at rest on the symmetry axis has the expression

$$J^{\mu}:\left(\frac{q}{r_0^2+a^2}\delta(r-r_0)\delta(\cos\theta-1),0,0,0\right). \tag{29}$$

The source term $T_1\Sigma$ can be calculated from (29). We obtain

$$T_1\Sigma=\frac{q}{\sqrt{2}}\frac{1}{r_0+\mathrm{i}a}\delta(r-r_0)\frac{\partial}{\partial\theta}\delta(\cos\theta-1). \tag{30}$$

We have shown (Linet 1977, unpublished) that the solution of (7) for the source (30) is

$$\phi_0=\frac{q}{\sqrt{2}}\frac{\Delta(r_0)}{r_0+\mathrm{i}a}\frac{1}{\Re^3}, \tag{31}$$

where we have introduced the notation

$$\Re^2 = (r-M)^2+(r_0-M)^2-2(r-M)(r_0-M)\cos\theta-(M^2-a^2)\sin^2\theta.$$

By integrating (8) for $\phi_0$ given by (31) we obtain the Debye potential

$$\bar{\psi}(r,\theta)=-\frac{1}{\sqrt{2}}\frac{q}{r_0+\mathrm{i}a}\frac{1}{\sin\theta}[\Re-(r-M)+(r_0-M)\cos\theta]-\frac{q}{\sqrt{2}}\frac{1-\cos\theta}{\sin\theta}. \tag{32}$$

By differentiation of (32) we find the tetrad components of the electromagnetic potential:

$$A(r,\theta)=q\frac{a}{r_0^2+a^2}\frac{1}{\sin\theta}\left(-\mathfrak{R}+(r-r_0\cos\theta)\frac{r-M-(r_0-M)\cos\theta}{\mathfrak{R}}-M(1-\cos\theta)\right)$$

$$A_t(r,\theta)=q\frac{1}{\Delta(r)(r^2+a^2)}\left((r_0r+a^2\cos\theta)\frac{(r-M)(r_0-M)-(M^2-a^2)\cos\theta}{\mathfrak{R}}\right.$$
$$\left.-rr_0(r_0-M)-a^2(r-M)+a^2\mathfrak{R}\right)+q\frac{r}{\Delta(r)}. \tag{33}$$

It is easy to see that (33) coincides with the expression for the components $A_t$ and $A_\phi$ given by Léauté (1977). The components of the electromagnetic field have also been given by Misra (1977).

### 4.2. *Charged current loop*

We consider a charged current loop in the equatorial plane. The Kerr space–time represents a rotating geometry; thus the interpretation of the components of the current density $J^\mu$ is difficult. We choose to define the current density with respect to the locally non-rotating observer $\eta$ introduced by Bardeen (1970). The components of $\eta^\mu$ are proportional to

$$(1,0,0,-g_{\phi t}/g_{\phi\phi}). \tag{34}$$

In the case of a loop of charge we define the current density as collinear to $\eta$. If $q$ is the total charge of the loop, we have

$$J^\mu:\left(\frac{q}{2\pi r_0^2}\delta(r-r_0)\delta(\cos\theta),0,0,-\frac{q}{2\pi r_0^2}\frac{g_{\phi t}}{g_{\phi\phi}}\delta(r-r_0)\delta(\cos\theta)\right). \tag{35}$$

For the stationary observer with respect to infinity there is a current loop with fictitious intensity

$$J_f=-\frac{g_{\phi t}}{g_{\phi\phi}}\bigg|_{\theta=\pi/2,\,r=r_0} q.$$

In the case of a current loop we define the current density as orthogonal to the observer $\eta$; then

$$J^\mu:\left(0,0,0,\frac{J}{2\pi r_0^2}\delta(r-r_0)\delta(\cos\theta)\right). \tag{36}$$

In (36) we must express $J$ from the intensity of the current I defined by the observer $\eta$. In the orthonormal tetrad associated with $\eta$ there is a three-dimensional current density

$$I^i:\left(0,0,\sqrt{-g_{\phi\phi}}\frac{J}{2\pi r_0^2}\delta(r-r_0)\delta(\cos\theta)\right).$$

In order to obtain the intensity $I$ of this current, we must integrate the density $I^i$ on the plane $t$ and $\phi$ constant:

$$I=\int I^3(\Sigma/\Delta^{1/2}(r))\,\mathrm{d}r\,\mathrm{d}\theta.$$

Consequently this integral expresses $J$ in terms of $I$:

$$J=\frac{\Delta^{1/2}(r_0)}{\sqrt{-g_{\phi\phi}|_{\theta=\pi/2,r=r_0}}}I. \tag{37}$$

For a charged current loop the source term $T_1\Sigma$ can be calculated.

$$\begin{aligned} T_1\Sigma=-\frac{1}{2\pi\sqrt{2}r_0}\Big(&\mathrm{i}[J+J_{\mathrm{f}})(r_0^2+a^2)-aq]\frac{\partial}{\partial r_0}\delta(r-r_0)\delta(\cos\theta)\\ &+[-a(J+J_{\mathrm{f}})+q]\delta(r-r_0)\frac{\partial}{\partial\theta_0}\delta(\cos\theta-\cos\theta_0)|_{\theta_0=\pi/2}\\ &-\mathrm{i}r_0(J+J_{\mathrm{f}})\delta(r-r_0)\delta(\cos\theta)\Big). \end{aligned} \tag{38}$$

Taking into account the expression for $J$ given by (37) and the definition of $J_{\mathrm{f}}$, the first term in (38) contains $\Delta^{1/2}(r_0)$ as a factor. Thus the $\phi_0$ found by convolution (22) with $G_1$ vanishes when the radius of the loop tends to the horizon and the electromagnetic field tends to the monopole field with charge $q$.

We are not going to determine explicitly the Debye potential corresponding to this charged current loop. Rather, we shall examine the physical problem of the charge accretion that one has studied with this source. In order to do this calculation we have to know the electrostatic potential $A_t$ on the symmetry axis (Wald 1974, Carter 1973). By equations (11) we see that

$$A_t(r,0)=-2\sqrt{2}\,\mathrm{Re}\left(\frac{1}{r-\mathrm{i}a}\frac{\bar{\psi}(r,\theta)}{\sin\theta}\bigg|_{\theta=0}\right). \tag{39}$$

Thus from equation (26) we find for a stationary axisymmetric source

$$\begin{aligned} A_t(r,0)=-2\pi\sqrt{2}\,\mathrm{Re}\bigg[\frac{1}{r-\mathrm{i}a}\Big(&\frac{1}{\sin\theta_0}[(r-M)^2+(r_0-M)^2-2(r-M)(r_0-M)\cos\theta_0\\ &-(M^2-a^2)\sin^2\theta_0]^{1/2}-(r-M)+(r_0-M)\cos\theta_0\Big)*T_1\Sigma\bigg]+E\frac{r}{r^2+a^2}. \end{aligned} \tag{40}$$

On the horizon the electrostatic potential (40) takes the value

$$A_t(r_+,0)=-\frac{\pi\sqrt{2}}{M}\mathrm{Re}\left[\left(1+\mathrm{i}\frac{a}{r_+}\right)\left((r_0-r_+)\frac{1+\cos\theta_0}{\sin\theta_0}\right)*T_1\Sigma\right]+\frac{E}{2M}. \tag{41}$$

Equation (41) is equivalent to that of Linet (1977b).

For a charged current loop we obtain from (41)

$$A_t(r_+,0)=-(J+J_{\mathrm{f}})a/r_0+q/r_0+Q/2M, \tag{42}$$

where $Q$ is the charge resulting from the accretion. In order that the total charge is zero, we put $q=-Q$. When the process of accretion is finished, we have $A_t(r_+,0)=0$. Taking into account the expressions for $J$ and $J_{\mathrm{f}}$, we find

$$Q=I\frac{2Ma}{r_0}\frac{1}{\Delta^{1/2}(r_0)}\left(r_0^2+a^2+\frac{2a^2M}{r_0}\right)^{1/2}. \tag{43}$$

The value $Q$ given by (43) is finite at the ergosphere $r_0=2M$ at $\theta_0=\pi/2$. With our definition of the current density $J^\mu$ we get rid of the infinity at $r_0=2M$ in (42) (equation given previously by Damour (1977) and Linet (1977b)). Equation (43) is a rectification of that of Petterson (1975). We note that Znajek (1978) has studied the charge accretion for a charged current loop situated axisymmetrically but not equatorially with respect to the Kerr metric.

## 5. Conclusion

In the Kerr metric we have determined the Debye potential corresponding to a stationary axisymmetric distribution of charges and currents. It should be mentioned that this method can make a numerical approach easier.

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